ECON 410: Recitation #3 — Answer Key

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1. **Classify the following utility functions by the shape of their indifference curves.**

Convex indifference curves:
U=F$^{1/2}C^{1/2}$, $U = B^{1/2}C^{2/3}$, $U = X^2Y^2$, $U = F^{1/3}C^{1/3}$,
$U = X^aY^b$ $(a > 1, b > 1)$, $U = \frac{3}{2}F^{1/10}C^{1/3}$, $U = X^{1/3}Y^{2/3}$,
$U = FC$, $U = \frac{4}{5}X^{1/2}Y^{2/3}$, $U = F^{0.25}C^{0.75}$,
$U = \ln X + \ln Y$, $U = (X^{1/2} + Y^{1/2})^2$, $U = (X^3 + Y^3)^{1/3}$.

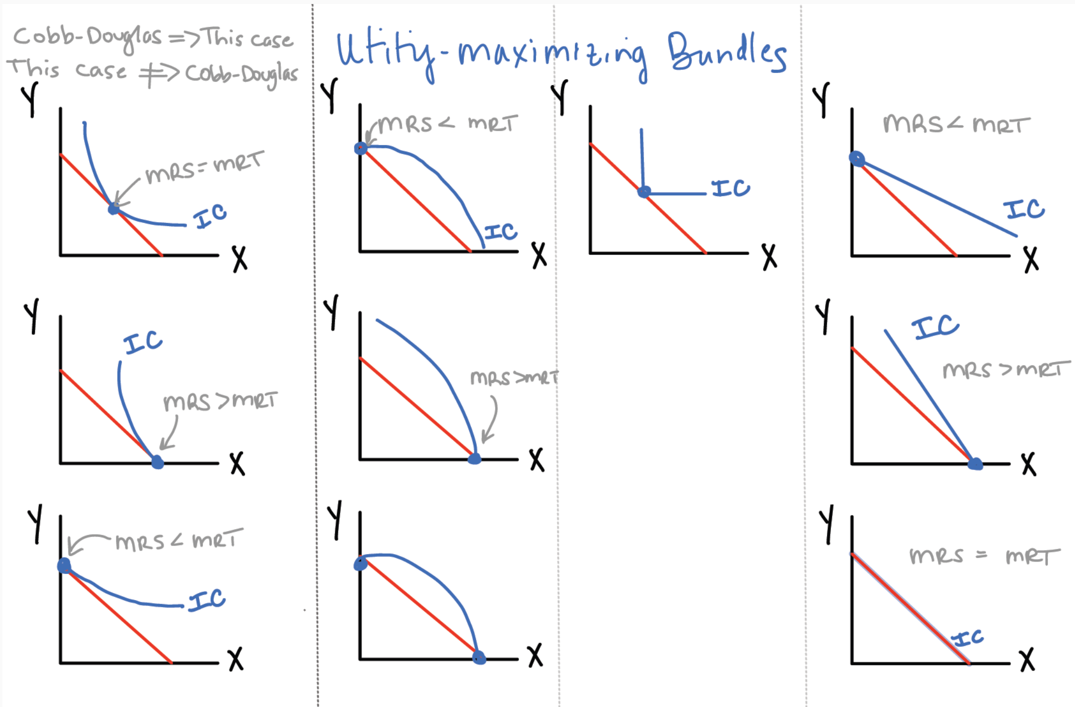
Concave indifference curves:
U=X+Y^2, $U = X^2 + Y^2$, $U = X^2 - Y^{1/2}$, $U = 4XY + X$.

Perfect complements:
U=min$(2B, 5T)$, $U = \min(3X, 5Y)$, $U = \min(B, 2T)$, $U = \min(4T, H)$.

Perfect substitutes:
U=2S+3A, U=10S+3M, U=X+3Y, U=5H+S.

2. **For each type of preference, illustrate how indifference curves intersect a budget constraint and identify the optimal bundle.**

Illustration of utility-maximizing bundles under different preference types.



Interpretation.

- **Convex indifference curves:** An interior solution occurs when the budget line is tangent to an indifference curve (MRS = MRT). Corner solutions arise when the tangency would lie outside the feasible set.
- **Concave indifference curves:** The optimum is always a corner solution.
- **Perfect complements:** The optimum occurs at the kink of the indifference curve.
- **Perfect substitutes:** The optimum is a corner solution unless the indifference curve is parallel to the budget line, in which case multiple optima exist.

3. **Four preference exercises.**

(1) **Sarah consumes waffles (W) and syrup (S). She requires exactly 4 teaspoons of syrup per waffle. Prices are $p_W = 1$, $p_S = 0.25$, and income is 34.**

Solution. Preferences are perfect complements:

$$U(W, S) = \min(W, S/4).$$

Optimal consumption satisfies $S = 4W$. Budget:

$$W + 0.25S = 34 \Rightarrow 2W = 34.$$

$$W^* = 17, \qquad S^* = 68.$$

(2) Conrad consumes coffee (C) and sugar (S). He needs 2 teaspoons of sugar per $1/3$ cup of coffee. Prices are $p_C = 1$, $p_S = 0.25$, income 20.

Solution. This implies $S = 6C$:

$$U(C,S) = \min(C, S/6).$$

Budget:

$$C + 0.25S = 20 \Rightarrow 2.5C = 20.$$

$$C^* = 8, \qquad S^* = 48.$$

(3) Matthew consumes sour cream (S) and buttermilk (B). He is willing to trade 1 cup of sour cream for $3/4$ cup of buttermilk. Prices are $p_S = 0.5$, $p_B = 1$, income 15.

Solution. Perfect substitutes:

$$U(S,B) = \tfrac{3}{4}S + B.$$

Bang-per-buck:

$$\frac{MU_S}{p_S} = 1.5 > \frac{MU_B}{p_B} = 1.$$

$$S^* = 30, \qquad B^* = 0.$$

(4) Grace consumes coffee (C) and tea (T). She is willing to trade 2 cups of coffee for 5 cups of tea. Prices are $p_C = 4$, $p_T = 1$, income 12.

Solution. Perfect substitutes:

$$U(C,T) = 5C + 2T.$$

Bang-per-buck:

$$\frac{MU_T}{p_T} = 2 > \frac{MU_C}{p_C} = 1.25.$$

$$T^* = 12, \qquad C^* = 0.$$