ECON 410: Recitation #2 — Solutions

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1. Andrew's preferences over electricity (E) and solar power (S) are

$$U(S,E) = \left(S^{1/2} + E^{1/2}\right)^2,$$

income $m = 120$, prices $p_E = 2$, $p_S = 4$.

(a) Use the Lagrangian method to calculate the utility-maximizing bundle.

Budget constraint:

$$4S + 2E = 120.$$

Lagrangian:

$$\mathcal{L}(S,E,\lambda) = \left(\sqrt{S} + \sqrt{E}\right)^2 + \lambda\left(120 - 4S - 2E\right).$$

Compute marginal utilities:

$$\frac{\partial U}{\partial S} = 2(\sqrt{S} + \sqrt{E}) \cdot \frac{1}{2\sqrt{S}} = \frac{\sqrt{S} + \sqrt{E}}{\sqrt{S}} = 1 + \sqrt{\frac{E}{S}},$$

$$\frac{\partial U}{\partial E} = 2(\sqrt{S} + \sqrt{E}) \cdot \frac{1}{2\sqrt{E}} = \frac{\sqrt{S} + \sqrt{E}}{\sqrt{E}} = 1 + \sqrt{\frac{S}{E}}.$$

FOCs:

$$1 + \sqrt{\frac{E}{S}} = 4\lambda, \qquad 1 + \sqrt{\frac{S}{E}} = 2\lambda.$$

Divide the first by the second:

$$\frac{1 + \sqrt{E/S}}{1 + \sqrt{S/E}} = \frac{4\lambda}{2\lambda} = 2.$$

Let $t = \sqrt{E/S}$, so $\sqrt{S/E} = 1/t$. Then

$$\frac{1+t}{1+1/t} = \frac{1+t}{(t+1)/t} = t.$$

So the tangency condition implies

$$t = 2 \quad \Rightarrow \quad \sqrt{\frac{E}{S}} = 2 \Rightarrow \frac{E}{S} = 4 \Rightarrow E = 4S.$$

Use the budget:

$$4S + 2(4S) = 12S = 120 \Rightarrow S^* = 10, \qquad E^* = 40.$$

(b) **Subsidy: for every unit of solar power Andrew buys, he receives \$2 (so p_S effectively becomes 2). Derive the new optimum using tangency.**

With the subsidy, prices are $p_S = 2$ and $p_E = 2$. The tangency condition is

$$\text{MRS}_{S,E} = \frac{MU_S}{MU_E} = \frac{p_S}{p_E} = 1.$$

From part (a), $\text{MRS}_{S,E} = \sqrt{E/S}$. Hence

$$\sqrt{\frac{E}{S}} = 1 \Rightarrow E = S.$$

New budget:

$$2S + 2E = 120 \Rightarrow 2S + 2S = 120 \Rightarrow S^* = 30, \qquad E^* = 30.$$

(c) **Lump-sum transfer: government gives Andrew \$60 (income becomes $m = 180$). What bundle should he now choose?**

A lump-sum transfer changes income only, not relative prices. Thus the tangency condition is unchanged from part (a):

$$\sqrt{\frac{E}{S}} = \frac{p_S}{p_E} = \frac{4}{2} = 2 \Rightarrow E = 4S.$$

New budget:

$$4S + 2E = 180.$$

Substitute $E = 4S$:

$$4S + 2(4S) = 12S = 180 \Rightarrow S^* = 15, \qquad E^* = 60.$$

(d) **Compute utils from each bundle and rank the bundles.**

Utility is

$$U(S,E) = (\sqrt{S} + \sqrt{E})^2.$$

(a) Baseline bundle $(S,E) = (10,40)$**:**

$$U_a = (\sqrt{10} + \sqrt{40})^2 = (\sqrt{10} + 2\sqrt{10})^2 = (3\sqrt{10})^2 = 9 \cdot 10 = 90.$$

(b) Subsidy bundle $(S,E) = (30,30)$**:**

$$U_b = (\sqrt{30} + \sqrt{30})^2 = (2\sqrt{30})^2 = 4 \cdot 30 = 120.$$

(c) Lump-sum bundle $(S,E) = (15,60)$**:**

$$U_c = (\sqrt{15} + \sqrt{60})^2 = (\sqrt{15} + 2\sqrt{15})^2 = (3\sqrt{15})^2 = 9 \cdot 15 = 135.$$

Therefore the ranking is:

$$(c)\ 135\ >\ (b)\ 120\ >\ (a)\ 90.$$

(e) On one graph with S on the x-axis and E on the y-axis, depict the utility-maximizing bundles from (a), (b), (c) using budget lines and indifference curves.

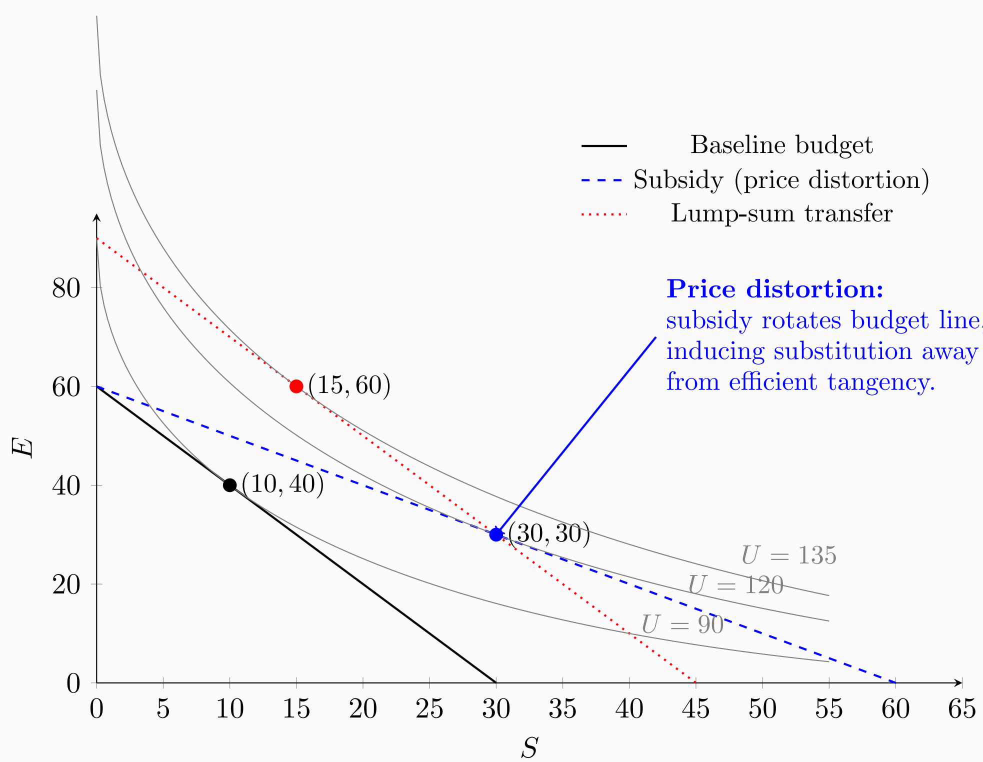
Budget lines (solve for E):

- (a) Baseline: $4S + 2E = 120 \Rightarrow E = 60 - 2S$ (slope -2).
- (b) Subsidy: $2S + 2E = 120 \Rightarrow E = 60 - S$ (slope -1).
- (c) Lump-sum: $4S + 2E = 180 \Rightarrow E = 90 - 2S$ (slope -2).

Optimal bundles:

$$(a) : (S, E) = (10, 40), \qquad (b) : (30, 30), \qquad (c) : (15, 60).$$

Graph.



Welfare comment. Even if the government spends the *same* amount under a subsidy as under a lump-sum transfer (or raises the same revenue under a distortionary tax as under a lump-sum tax), the *price distortion* is typically more harmful because it changes *relative prices* and forces a wedge between MRS and the true marginal rate of transformation:

$$\text{Lump-sum:} \quad \text{MRS} = \frac{p_S}{p_E} \text{ (no wedge)} \quad \text{vs.} \quad \text{Distortionary price:} \quad \text{MRS} = \frac{\tilde{p}_S}{p_E} \neq \frac{p_S}{p_E}.$$

A lump-sum policy shifts the budget set outward/inward without changing its slope (no substitution distortion), whereas a subsidy/tax rotates the budget line (creates substitution distortion). Hence, *for the same fiscal cost/revenue*, the distortionary price policy typically yields *lower utility* than the lump-sum policy.