ECON 410: Recitation #1 — Solutions

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January 9, 2026

1. **Simplify the following expression as much as possible:**

$$\left(\frac{(1/2)\, x^{-1/2} y}{(1/3)\, x^{1/2} y^{1/2}} + y^{1/2} \right)^2.$$

Solution.

$$\frac{(1/2)\, x^{-1/2} y}{(1/3)\, x^{1/2} y^{1/2}} = \frac{1/2}{1/3}\, x^{-1/2-1/2}\, y^{1-1/2} = \frac{3}{2}\, x^{-1}\, y^{1/2} = \frac{3y^{1/2}}{2x}.$$

$$\Rightarrow \quad \left(\frac{3y^{1/2}}{2x} + y^{1/2} \right)^2 = \left(y^{1/2} \left(\frac{3}{2x} + 1 \right) \right)^2 = \left(y^{1/2} \cdot \frac{3+2x}{2x} \right)^2 = y \cdot \frac{(3+2x)^2}{4x^2}.$$

$$\boxed{\left(\frac{(1/2)\, x^{-1/2} y}{(1/3)\, x^{1/2} y^{1/2}} + y^{1/2} \right)^2 = \frac{y(3+2x)^2}{4x^2}}$$

2. **Use the following expression to solve for x:**

$$\frac{2x^{-1/3} y^{1/2}}{4x^{1/3} y^{-1/2}} = \frac{a}{b}.$$

Solution.

$$\frac{2x^{-1/3} y^{1/2}}{4x^{1/3} y^{-1/2}} = \frac{2}{4}\, x^{-1/3-1/3}\, y^{1/2-(-1/2)} = \frac{1}{2}\, x^{-2/3}\, y = \frac{y}{2x^{2/3}}.$$

So

$$\frac{y}{2x^{2/3}} = \frac{a}{b} \quad \Longrightarrow \quad by = 2a\, x^{2/3} \quad \Longrightarrow \quad x^{2/3} = \frac{by}{2a}.$$

Raise both sides to the power $3/2$:

$$\boxed{x = \left(\frac{by}{2a} \right)^{3/2}}$$

3. **Partially differentiate the following function with respect to x and z:**

$$f(x, z, q) = \frac{3x}{qz^2} - x^{1/2}q.$$

Solution. (Treat q as a constant.)

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\frac{3x}{qz^2}\right) - \frac{\partial}{\partial x}\left(x^{1/2}q\right) = \frac{3}{qz^2} - \frac{1}{2}x^{-1/2}q.$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left(\frac{3x}{q}z^{-2}\right) = \frac{3x}{q}\cdot(-2)z^{-3} = -\frac{6x}{qz^3}.$$

$$\boxed{\frac{\partial f}{\partial x} = \frac{3}{qz^2} - \frac{q}{2\sqrt{x}}, \qquad \frac{\partial f}{\partial z} = -\frac{6x}{qz^3}}$$

4. **Partially differentiate the following function with respect to x and y. Then form the ratio $\dfrac{\partial f/\partial x}{\partial f/\partial y}$ and simplify:**

$$f(x, y) = 2x^{4/3}y^{1/3}.$$

Solution.

$$\frac{\partial f}{\partial x} = 2\cdot\frac{4}{3}x^{4/3-1}y^{1/3} = \frac{8}{3}x^{1/3}y^{1/3}, \qquad \frac{\partial f}{\partial y} = 2x^{4/3}\cdot\frac{1}{3}y^{1/3-1} = \frac{2}{3}x^{4/3}y^{-2/3}.$$

Thus,

$$\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = \frac{\frac{8}{3}x^{1/3}y^{1/3}}{\frac{2}{3}x^{4/3}y^{-2/3}} = \left(\frac{8}{3}\right)\left(\frac{3}{2}\right)x^{1/3-4/3}\,y^{1/3-(-2/3)} = 4\,x^{-1}\,y = \frac{4y}{x}.$$

$$\boxed{\frac{\partial f/\partial x}{\partial f/\partial y} = \frac{4y}{x}}$$